Finite State Machines

The Simplest Model of Computation

What is a Finite State Machine?

- Mathematical model of Computation
- Abstract Machine
- Is in exactly one state at any given time
- Changes state based on input
- Surprisingly flexible
- Recognizes a Language

Practical examples:

- Vending machines
- Elevators
- Traffic signals
- Combination locks
- Antikythera mechanism
- Automatons
  - http://youtu.be/bY_wfKVjuJM

Are Robots FSMs? Why or why not?

FSM Characteristics

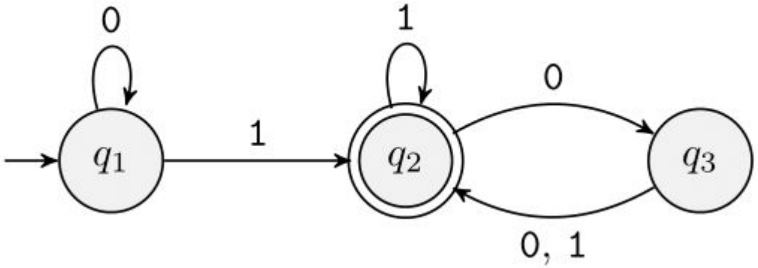
Limited Memory

- Small Computer
- Microcontroller

Finite *(It's in the name!)*

Family of:

- Regular Languages
- Regular Expressions

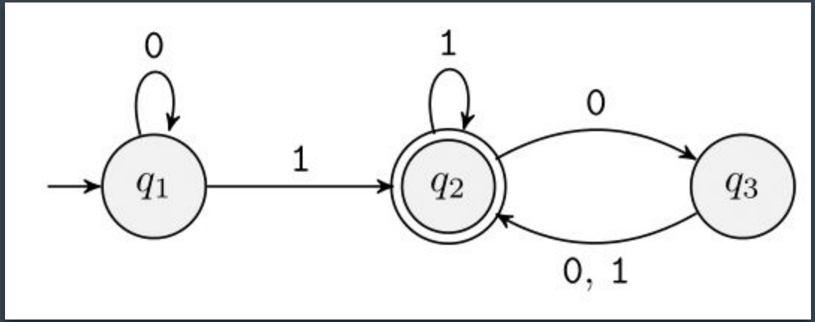


A picture is worth a thousand words...

Nodes = States
Edges = Transitions

Formal Definition of a Finite State Machine

$$M = (Q, \Sigma, \delta, q_0, F)$$



Q Set of states (finite)

Σ Alphabet of symbols (finite)

δ The transition function
$\delta: Q \times \Sigma \rightarrow Q$

q_0 The starting (initial) state
$q_0 \in Q$

F The set of "Accept" states
$F \subseteq Q$

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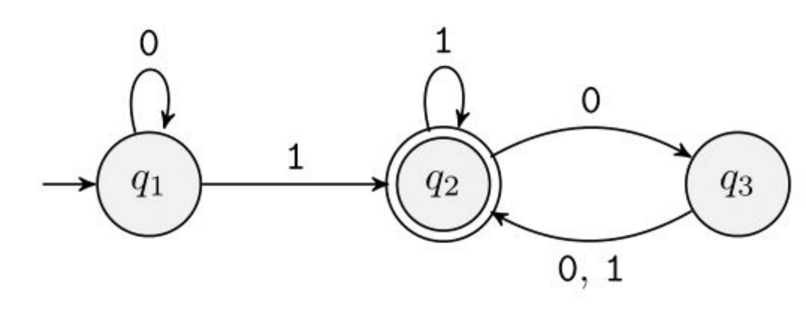


$$M = (\{q_1, q_2, q_3\}, \{0, 1\}, \delta, q_1, \{q_2\})$$

Q $\{q_1, q_2, q_3\}$

Σ $\{0, 1\}$

δ

	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2

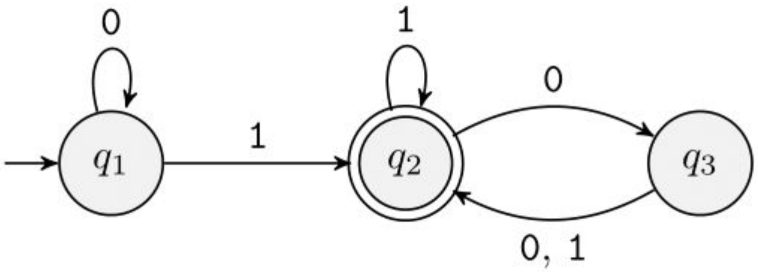
q_0 q_1

F $\{q_2\}$

FSM Use #1: Generating Strings

1. Begin at starting state
2. Take transitions at random
   Transitions are recorded, which is the string being generated
3. End only on valid states

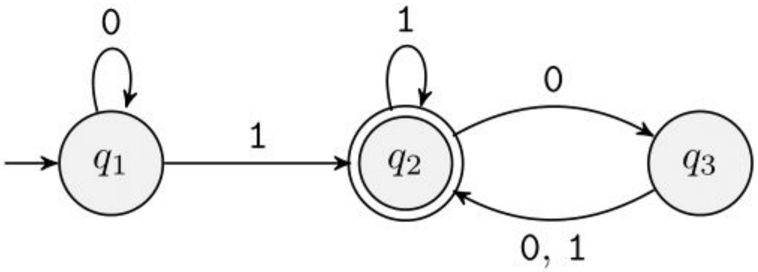
What is the set of strings that can be generated?



What Language will this FSM generate?

FSM Use #2: Accepting Strings

1. Begin at starting state
2. Start at the 1st symbol of the string
3. Follow transitions as determined by the symbol, 1 symbol per transition
4. Process ALL symbols in the string
5. Is the machine in a final state?



A string is either "accepted" or "rejected"

Other FSM Considerations

Empty strings

- ε
- Starting state is also an accept state

Empty Language

- $\emptyset = \{\}$
- There is no path from the starting state to any accept state

Important:

- $\varepsilon \neq \emptyset$
- $\{\varepsilon\} \neq \emptyset$

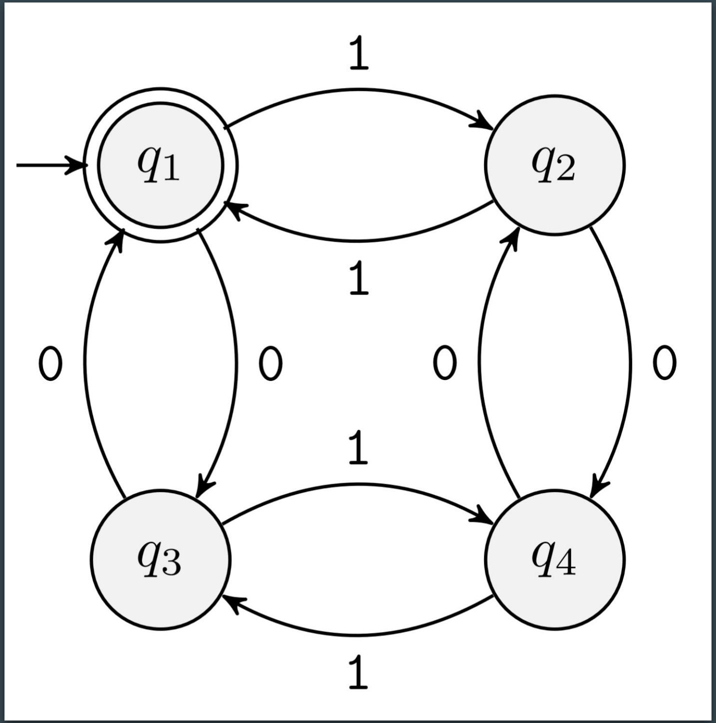
Dead states

- A state that exists as a "reject" state
- Often omitted from diagrams
- If an edge is omitted it is assumed to be a transition to the dead state
- Understood as being a sink node (no escape once reached)

Example

Construct a FSM that will not accept any string unless it has an even number of `0`s and `1`s, where $\Sigma = \{0, 1\}$.

What is its complement?



Formal Definition of Computation

Let $M = (Q, \Sigma, \delta, q_0, F)$

Let $w_1 w_2 \ldots w_n$ be a string w where $w_i \in \Sigma$

M accepts w if there is a sequence of states $r_0, r_1, r_2 \ldots r_n$ in Q such that:

1. $r_0 = q_0$,
2. $\delta(r_i, w_{i+1}) = r_{i+1}$ for $0 \leq i < n$, and
3. $r_n \in F$

M "recognizes" language A if

$A = \{w \mid M \text{ accepts } w\}$

We now have a tool that we can use to understand Regular Languages!